3. JIT• 27-7-10

Journal of Industrial Technology (25-32)

THE FLYING VEHICLE FLIGHT TRAJECTORY OPTIMIZATION (THREE-DIMENSIONAL MOTION)

V. P. Nesterov

School of Aerospace Engineering, Engineering Campus,
Universiti Sains Malaysia, Seri Ampangan,
14300 Nibong Tebal, Pulau Pinang, Malaysia
(vpnesterov@mail.ru)

RINGKASAN : *Satu teknik rekabentuk dicadangkan untuk perpindahan optima dari keadaan mula dan akhir di dalam ruang tiga dimensi, di tengah medan graviti, di dalam ruang kosong, dan di bawah pergerakan tujahan yang tetap. Kajian ini merangkumi algorisma kawalan pergerakan kenderaan yang dibina melalui Teori Gunaan Kawalan Optima. Perubahan sudut serangan* $\alpha(t)$ *dan sudut golekan halaju* $\gamma_V(t)$ *dibandingkan dengan masa penerbangan kenderaan ditunjukkan pada nilai kecondongan yang diberi. Variasi nilai penambahan kos untuk karakter halaju pada akhir penerbangan dan lebihan perubahan antara jisim akhir kenderaan terbang untuk kecondongan* $\Delta i \neq 0$ *dan untuk* $\Delta i = 0$ *bergantung kepada nilai kecondongan* Δi *di bawah kawalan optima (sudut serangan dan sudut halaju golekan) juga ditunjukkan.*

ABSTRACT : A design technique is suggested for the flying vehicle optimal transfer from the initial to a final state in three-dimensional space, in the central gravitation field, in empty space and under the action of fixed thrust. The given study considers the flying vehicle motion control algorithm built proceeding from the optimal control applied theory. The changes of the angle of attack $\alpha(t)$ and velocity roll angle $\gamma_V(t)$ versus the flying vehicle flight time are presented for the given inclination value. A value variation of additional costs of the characteristic velocity at the end of flight and a residue change between the flying vehicle final mass for inclination $\Delta i \neq 0$ and for $\Delta i = 0$ depending on inclination value Δi under optimal control (the angle of attack and velocity roll angle) are also shown.

KEYWORDS : Three-dimensional motion, optimal control, flying vehicle, equations of flying vehicle motion, angle of attack, velocity roll angle

INTRODUCTION

Nesterov (2009) suggested a design procedure for the flying vehicle (FV) optimal transfer from the initial to a final state in one plane, in the central gravitation field, in empty space and under the action of fixed thrust. FV control is performed by the angle of attack. In a given study, a design procedure for the FV optimal transfer from the initial to a final state in three-dimensional space is proposed. FV control is performed by the angle of attack and velocity roll angle.

METHODOLOGY

The present study considers the design technique of the flying vehicle (FV) optimal transfer under the action of constant thrust exceeding the Newtonian force, from initial to final state, characterized by prescribed values of FV radius-vector relative to the attracting centre, velocity, angle of inclination of the velocity vector to a local horizon and trajectory inclination.

The following assumptions were made:

(i) the Earth is of a spherical shape and during the transfer phase it does not rotate,

(ii) its gravity field is central,

(iii) the FV flight runs at high altitudes where air resistance is negligible and

(iv) the FV control system is inertialess.

The algorithm analysis of FV optimal transfer trajectory is built based on the applied theory of optimal control (Bryson & Ho, 1969), (Nesterov, 2009).

The equations of FV motion in its centric rectangular vertical-wind-body coordinate system are used according to Gorbatenko *et al.* (1969)

$$\overset{\circ}{\bar{x}} = \bar{f}(\bar{x},\bar{u},t). \qquad \text{(7 equations)} \qquad (1)$$

$$\left.\begin{aligned}
&\frac{dV}{dt} = \frac{P}{m}\cos\alpha - g_0\left(\frac{R_0}{R}\right)^2 \sin\theta;\\
&\frac{d\theta}{dt} = \frac{P}{mV}\sin\alpha\cos\gamma_V - g_0\left(\frac{R_0}{R}\right)^2 \frac{\cos\theta}{V} + \frac{V}{R}\cos\theta;\\
&\frac{d\psi_V}{dt} = -\frac{P}{mV}\frac{\sin\alpha\sin\gamma_V}{\cos\theta} + \frac{V}{R}\cos\theta \times \mathrm{tg}\varphi \times \sin\psi_V;\\
&\frac{dR}{dt} = V\sin\theta;\\
&\frac{d\varphi}{dt} = \frac{V}{R}\cos\theta\cos\psi_V;\\
&\frac{d\lambda'}{dt} = -\frac{V}{R}\cos\theta\sin\psi_V\sec\varphi;\\
&\frac{dm}{dt} = -\overset{\circ}{m}{}^a;
\end{aligned}\right\} \tag{2}$$

where $\bar{x} = (V, \theta, \psi_V, R, \varphi, \lambda', m)^T$ is the FV current state vector, $\bar{f}(\bar{x}, \bar{u}, t)$ is the vector of the right sides of the system of differential equations of vehicle's motion, t is the current time of the flight, V is the vehicle's velocity, θ is the velocity vector angle of inclination to a local horizon, ψ_V is the velocity heading angle, R is the value of FV radius-vector relative to the attracting centre, φ is the geocentric latitude, λ' is the geocentric longitude, m is the FV mass, P is the FV thrust value, g_0 is the free-fall acceleration value on Earth's surface, $g = g_0\left(\frac{R_0}{R}\right)^2$ is the free-fall acceleration value at vehicle's attitude, R_0 is the Earth's radius, $\overset{\circ}{m}{}^a$ is the absolute value of FV mass flow, α is the angle of attack and γ_V is the velocity roll angle.

For parameters of FV motion control we use the angle of attack and the velocity roll angle:

$$\bar{u}(t) = (\alpha(t), \gamma_V(t))^T. \tag{3}$$

The FV initial state vector is prescribed:

$\bar{x}(t_0)$, (7 initial conditions). (4)

The FV terminal state is confined as

$$\bar{\psi} = [\,\bar{x}_1(t_f), t_f\,] = 0, \tag{5}$$

where t_f is the current time at the end of flight,

$\bar{x}_1(t_f) = (V, \theta, \psi_V, R, \varphi, \lambda')^T$, $\bar{\psi} = (\psi_1, \psi_2, \psi_3, \psi_4)^T$, (4 conditions),

where $\psi_1 = V - V_f$ - by the velocity vector value at the end of transfer leg, $\psi_2 = \theta - \theta_f$ – by the value of inclination angle of velocity vector to a local horizon at the end of the transfer leg, $\psi_3 = R - R_f$ – by FV radius-vector value relative to the attracting centre at the end of the transfer leg, $\psi_4 = i - i_f$ – by the inclination value i.

The negative final mass is the FV performance criterion:

$$-\,m(t_f)\,. \tag{6}$$

So, the task is to define such a control $\bar{u}\,(t)$, which could provide the minimum of negative value of final mass [- $m(t_f)$] with available differential constraints (1) and limitations to a terminal state (5). The Hamiltonian H (Bryson & Ho, 1969) for this task can be written as:

$$H(\bar{x},\bar{u},\bar{\lambda},t) = \lambda_1 \frac{dV}{dt} + \lambda_2 \frac{d\theta}{dt} + \lambda_3 \frac{d\psi_V}{dt} + \lambda_4 \frac{dR}{dt} + \lambda_5 \frac{d\varphi}{dt} + \lambda_6 \frac{d\lambda'}{dt} + \lambda_7 \frac{dm}{dt}, \tag{7}$$

where $\bar{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7)^T$ - the Lagrangian multiplier vector (function interference vector upon the functional).

The Euler- Lagrange set of equations looks like:

$$\mathring{\lambda} = -\left(\frac{\partial H}{\partial \bar{x}}\right)^T, \qquad \text{(7 equations)}, \tag{8}$$

$$\frac{\partial H}{\partial \bar{u}} = 0, \qquad \text{(2 equations)}, \tag{9}$$

Terminal conditions give us the equalities:

$$\bar{\lambda}^T(t_f) = \left(\frac{\partial \Phi}{\partial \bar{x}}\right)_{t=t_f}, \qquad \text{(7 conditions)}, \tag{10}$$

where $\Phi(\bar{x},\bar{v},t) = -m(t) + \bar{v}^{\mathrm{T}}\bar{\psi}(\bar{x},t), \bar{v} = (v_1,v_2,v_3,v_4)^{\mathrm{T}}$, (11)

$$\Omega[\bar{x},\bar{u},\bar{v},t]_{t=t_f} \equiv \left(\frac{d\Phi}{dt}\right)_{t=t_f} = 0, \quad \text{(1 condition)} \quad (12)$$

Time of transfer ending t_f is determined implicitly by means of terminal boundary conditions (10).

From equations (7) and (8), we can write down:

$$\overset{\circ}{\bar{\lambda}}{}^{\mathrm{T}} = -\bar{\lambda}^{\mathrm{T}}\frac{\partial\bar{f}}{\partial\bar{x}}. \quad (13)$$

Thus, it is required to find a solution of the system of 14 differential equations (1) (set of equations of flying vehicle motion) and (8) (set of equations of costate variables) and determine 5 values of unknown parameters $\bar{v}$ and t_f so that to meet seven initial conditions (4) and twelve terminal conditions (5), (10), (12). Determination of $\bar{u}(t)$ is made using the equations (9):

$$\gamma_{V1} = \mathrm{arctg}\left[-\frac{\lambda_3}{\lambda_2\cos\theta}\right], (-\pi \le \gamma_V \le \pi),$$
$$\gamma_{V2} = \gamma_{V1} + \pi. \quad (14)$$

For each value γ_V we determine two values of the angle of attack:

$$\alpha_1 = \mathrm{arctg}[\frac{1}{\lambda_1 V}(\lambda_2\cos\gamma_V - \lambda_3\frac{\sin\gamma_V}{\cos\theta})], \quad (15)$$
$$\alpha_2 = \alpha_1 - \pi.$$

According to Bryson & Ho (1969), the time optimal control $\hat{\bar{u}}$ should minimize $H(\bar{u})$:

$$\hat{H}(\bar{x},\hat{\bar{u}},\bar{\lambda},t) \le H(\bar{x},\bar{u},\bar{\lambda},t) .$$

Solution of the equations (1) – (12) can be reduced to a solution of nine-parametric boundary-value equations (Smirnov & Nesterov, 1980) which is reduced to a finding of roots of a set of equations:

$$V_0 = V(\bar{\kappa}), \theta_0 = \theta(\bar{\kappa}), \psi_{V0} = \psi_v(\bar{\kappa}), R_0 = R(\bar{\kappa}), \varphi_0 = \varphi(\bar{\kappa}),$$
$$\lambda'_0 = \lambda'(\bar{\kappa}), m_0 = m(\bar{\kappa}), i' = i(\bar{\kappa}), \Omega_g = \Omega(\bar{\kappa}), \quad (16)$$

where $\overline{\kappa} = (t_f, v_1, v_2, \psi_{vf}, v_3, \varphi_f, \lambda'_f, m_f, v_4)$ is a vector of unknown parameters. (17)

Here: $V_0, \theta_0, \psi_{V0}, R_0, \lambda'_0, m_0$ are given values of FV parameters to the time, t_0. They must be found when solving a boundary-value problem; i', Ω_g are given values of inclination and parameter Ω (from condition (12)).

Integration of the system of differential equations (1) and (13) are performed with a negative step until the time t_0, then we find the discrepancies by conditions (16). Solution of the boundary-value problem runs until the conditions (16) be met with a given accuracy. To solve the boundary-value problem we make use of the Newton method for solution of the system of nonlinear equations. In the case that we know well enough the initial approximations of Lagrangian multiplier vector $\overline{\lambda}(t_0)$, the solution of the problem (1)-(12) can be reduced to a finding of five roots of a set of equations:

$$\theta_f = \theta(\overline{\sigma}),\ R_f = R(\overline{\sigma}), i' = i(\overline{\sigma}), 0 = \Delta v_4(\overline{\sigma}), \Omega_g = \Omega(\overline{\sigma}), \qquad (18)$$

where $\overline{\sigma} = (\lambda_1(t_0), \lambda_2(t_0), \lambda_3(t_0), \lambda_4(t_0), \lambda_5(t_0))$ is a vector of unknown parameters (19)

Here, θ_f, R_f, i' are given values of FV parameters to the time moment t_f and inclinations to be performed in consequence of boundary-value problem solving, Ω_g is obtained from (12), Δv_4 is obtained from (10).

When taking into account that in (10) $\lambda_3(t_f) = v_4 \dfrac{\partial i}{\partial \psi_V}, \lambda_5(t_f) = v_4 \dfrac{\partial i}{\partial \varphi}$,

we get $\Delta v_4 = \dfrac{\lambda_3(t_f)}{(\partial i / \partial \psi_V)} - \dfrac{\lambda_5(t_f)}{(\partial i / \partial \varphi)}$.

In this case, integration of the system of differential equations (1) and (13) are performed with a positive step from the time, t_0 until the fulfilment of prescribed condition by velocity at the end of transfer $V = V_f$, and then we determine the residuals by conditions in (18). $V = V_f$ is the sixth condition in addition to condition (18), hence we solve a six-parametric boundary-value problem.

RESULTS

The given study considers the FV motion control algorithms built proceeding from the optimal-control applied theory. FV control is performed by angle of attack and velocity roll angle.

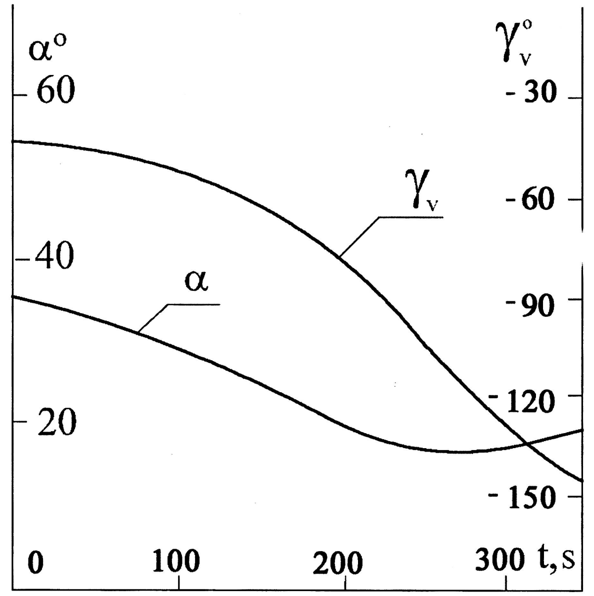

Figure 1. *The angle of attack and velocity roll angle variation*

Figure 1 illustrates the changes of the angle of attack $\alpha(t)$ and velocity roll angle $\gamma_V(t)$ depending on the FV flight time for $\Delta i = 25^{\circ}$.

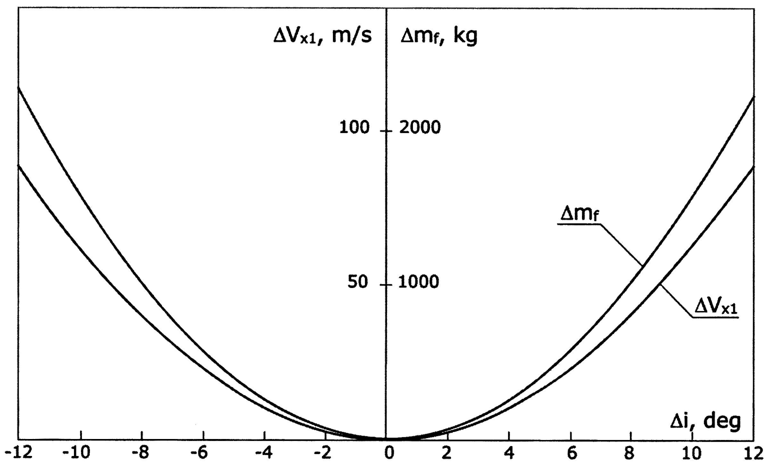

Figure 2. *Value of variation of additional costs of the characteristic velocity at the end of flight showing the dependence on the inclination value Δi under optimal control*

Figure 2 illustrates a value variation of additional costs of the characteristic velocity at the end of flight $(\Delta V_{x1} = V_{x1}(\Delta i) - V_{x1}(\Delta i = 0))$ and a residue change $(\Delta m_f = m_f(\Delta i) - m_f(\Delta i = 0))$ between the FV final mass for inclination $\Delta i \neq 0$ and for $\Delta i = 0$ depending on the inclination value Δi under optimal control (the angle of attack and velocity roll angle).

CONCLUSION

Thus, in this investigation a design procedure was suggested and realized in the software modules using the Fortran-IV algorithmic language for the flying vehicle optimal transfer from the initial to a final state in three-dimensional space, in the central gravitation field, in empty space and under the action of fixed thrust. This investigation reveals the changes of the angle of attack $\alpha(t)$ and velocity roll angle $\gamma_V(t)$ versus the flying vehicle flight time which are presented for the given inclination value. A value variation of additional costs of the characteristic velocity at the end of flight and a residue change between the flying vehicle final mass for inclination $\Delta i \neq 0$ and for $\Delta i = 0$ depending on inclination value Δi under optimal control are also shown.

REFERENCES

Bryson, A. E., Ho You-Chi. (1969). Applied optimal control. Blaisdell Publishing Company, London. pp 42-89.

Gorbatenko S.A., Makashov E.M., Polushkin Yu. F., Sheftel L. V. (1969). Flight mechanics. *Engineering Handbook*. Mashinostroenie, Moscow. pp 129, 188-189.

Nesterov V. P. (2009). The Flying Vehicle Optimum Flight Trajectory Design. *Journal of Industrial Technology*. **Vol. 18(1)**.: pp 33-40.

Smirnov O. L., Nesterov V. P. (1980). The Investigation of Guidance Algorithms During Landing A Vehicle with the Radio Beacon. *XXXI Congress of the International Astronautical Federation. Abstracts of Papers*. Tokyo, Japan. pp 248.